Summer Fields School
KAILASH COLONY, NEW DELHI-110048

38th ARYABHATTA INTER-SCHOOL MATHEMATICS COMPETITION: 2021

CLASS - XI

Instructions:

1. The duration of the Competition is 2 hours (10:30 am – 12:30 pm).
2. The Question Paper contains 75 questions. There are 50 questions of 1 mark each and 25 questions of 2 marks each. The marks are indicated as points (1 or 2) with each question.
3. All the questions are Multiple Choice Questions. You have to select the correct option.
4. You have to submit the Paper by clicking the 'Submit' button at the end of the question paper.
5. You have to submit the Paper by 12:30 pm.
6. You can submit the paper only once. No second chance will be provided.

SECTION-A Write the correct option (A, B, C or D) in the Answer sheet.

1. How many numbers are there from 101 to 999 such that at least one of the digits is 5?

(A) 647 (B) 288

(C) 252 (D) None of these

2. The number of the divisors of 72000 is

(A) 105 (B) 84

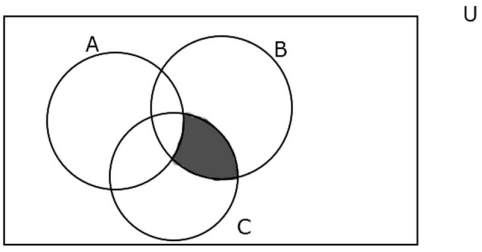
(C) 120 (D) None of these

3. The number of elements in the Power Set of the set
$\mathrm{A} = \{x : x \text{ is a zero of the polynomial } (3x^7 - 192x) \text{ and } x \in \mathrm{Z}\}$

(A) 8 (B) 64

(C) 16 (D) None of these

4. In the following Venn's Diagram, the shaded part represents:



(A) $A \cap \mathrm{C} \cap \mathrm{B}^c$ (B) $\mathrm{A}^c \cap \mathrm{C} \cup \mathrm{B}$

(C) $\mathrm{B} \cap \mathrm{C} \cap \mathrm{A}^c$ (D) None of these

5. The range of the function $f(x) = \dfrac{x^2 + x + 1}{x^2 - x + 1}$ is

(A) $\left(\dfrac{1}{2}, 2\right)$ (B) $\left[\dfrac{1}{3}, 3\right]$

(C) $\left[\dfrac{1}{2}, 2\right)$ (D) None of these

6. The value of $\cos\frac{\pi}{7}\cos\frac{3\pi}{7}\cos\frac{5\pi}{7}$ is equal to

(A) 2 (B) –2

(C) $\frac{1}{2}$ (D) None of these

7. $\sqrt{-\sqrt{-1}}$ is equal to

(A) $\frac{\sqrt{2}}{2}(1-i)$ (B) $\frac{\sqrt{2}}{2}(1+i)$

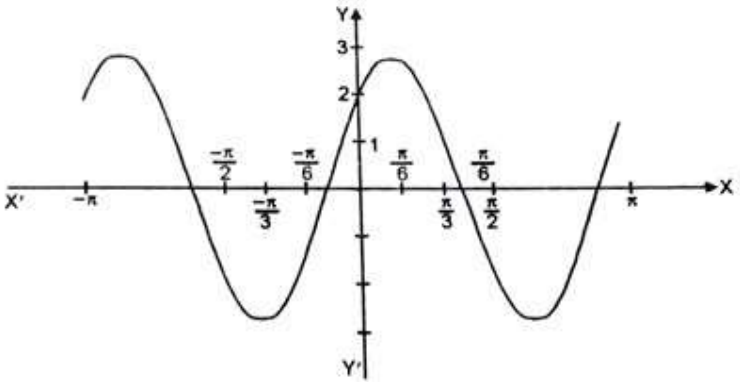
(C) $\frac{1}{2}(1-i)$ (D) None of these

8. The smallest value of x so that $\cos 3x - 3\cos 2x + \cos x = \sin 3x - 3\sin 2x + \sin x$ is

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{8}$ (D) None of these

9. The function represented by the following graph is



(A) $y = 3\sin\left(2x + \frac{\pi}{6}\right)$ (B) $y = \sin\left(2x + \frac{\pi}{3}\right)$

(C) $y = 2\cos\left(2x + \frac{\pi}{3}\right)$ (D) None of these

10. The solution set for the inequality $\frac{2-x}{5} - \frac{1-2x}{3} \geq \frac{3x-2}{4}, x \in R$ is

(A) $(-\infty, 3]$ (B) $(-\infty, 2]$

(C) $[3, \infty)$ (D) None of these

11. If $5\tan x + 3\sec x = 4$, the value of $5\sec x + 3\tan x$ is

(A) $3\sqrt{2}$ (B) $2\sqrt{2}$

(C) $4\sqrt{2}$ (D) None of these

12. The angle in radian measure between the hands of a clock at 1:20 pm is

(A) $\frac{2\pi}{9}$ (B) $\frac{2\pi}{5}$

(C) $\frac{\pi}{2}$ (D) $\frac{4\pi}{9}$

13. The number of words that can be formed by arranging the letters of the word 'DAUGHTER' such that all vowels are always together and the extreme letters of the words are D and R, is

(A) 248 (B) 144

(C) 288 (D) None of these

14. Which of the following is not true?

(A) ${}^{n}C_{r} = {}^{n}C_{n-r}$ (B) ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

(C) ${}^{n}P_{r} = r!\,{}^{n}C_{r}$ (D) None of these

15. The value of 'r' if the coefficients of $(3r+2)^{\text{th}}$ and $(r-4)^{\text{th}}$ terms in the expansion of $(1+x)^{28}$ are equal, is

(A) 12 (B) 8

(C) 7 (D) None of these

16. The approximate value of $(1.004)^{4}$ is

(A) 1.016 (B) 1.0016

(C) 1.16 (D) None of these

17. Three numbers, whose sum is 21 are in A.P. If 11, 9, and 7 be added to them respectively, then these numbers are in G.P. The smallest of these numbers is

(A) 7 (B) 5

(C) 6 (D) None of these

18. The term independent of x in the expansion of $\left(x^2 - \frac{3}{x}\right)^{24}$ is

(A) ${}^{24}C_8 3^{16}$ (B) ${}^{24}C_6 3^{18}$

(C) ${}^{24}C_7 3^{17}$ (D) None of these

19. The number of different ways in which the letters of the word 'MATHEMATICS' can be arranged such that all vowels occur together, are

(A) 122280 (B) 120960

(C) 162520 (D) None of these

20. If ${}^nC_{r+1} : {}^nC_r = 2:1,\ {}^nC_{r+2} : {}^nC_{r+1} = 3:2$ then the value of $\boldsymbol{n + r}$ is

(A) 22 (B) 24

(C) 18 (D) None of these

21. A team of 7 persons is to be formed from 9 boys and 4 girls. In how many ways can it be done if the team consists of at most 2 girls?

(A) 1128 (B) 1266

(C) 1816 (D) None of these

22. The sum of four numbers in G.P. is 105 and the arithmetic mean of the second and last number is 35. The largest number is

(A) 64 (B) 48

(C) 52 (D) None of these

23. The locus of the midpoint of a rod of length 4 units which slides between two perpendicular lines is

(A) $2x + 3y = 16$ (B) $4x^2 + y^2 = 1$

(C) $x^2 + y^2 = 4$ (D) None of these

24. The eccentricity of the ellipse passing through (3, 1) and (–2, 2), is

(A) $\sqrt{\frac{2}{5}}$ (B) $\frac{2}{\sqrt{5}}$

(C) $\frac{\sqrt{2}}{5}$ (D) None of these

25. The point of intersection of the line joining (5, 3, 1) and (1, –1, 5) with XZ-plane is

(A) (3, 0, –2)
(B) (2, 0, 4)
(C) (-3, 0, 4)
(D) None of these

26. $\lim_{x\to 1}\frac{x^3-3x+2}{x^6-6x+5}$ is equal to

(A) $\frac{1}{5}$
(B) $\frac{1}{3}$
(C) $\frac{2}{5}$
(D) None of these

27. The mean deviation about median for the data: 14, 13, 11, 16, 17, 18, 12, 10, 16, 13 is

(A) 1.92
(B) 1.98
(C) 2.2
(D) None of these

28. Out of 20 numbers from 5 to 24, three numbers are chosen. The probability that the chosen numbers are not consecutive, is

(A) $\frac{144}{145}$
(B) $\frac{172}{185}$
(C) $\frac{187}{190}$
(D) None of these

29. $\lim_{x\to 0}\frac{1-\cos x}{x^3\cot x}$ is equal to

(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) 1
(D) None of these

30. The probability that a leap year selected at random that do not contain either 53 Saturdays or 53 Sundays, is

(A) $\frac{1}{7}$
(B) $\frac{2}{7}$
(C) $\frac{3}{7}$
(D) None of these

31. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is equal to

(A) $\cos ec^2\left(\frac{\pi}{4} + x\right)$ (B) $co\sec^2\left(\frac{\pi}{4} - x\right)$

(C) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (D) None of these

32. The distance of the line $2x - 3y = 9$ from the point (1, 1) measured parallel to the line $x + y = 1$ is

A) $\sqrt{2}$ (B) $2\sqrt{2}$

(C) $\frac{3}{\sqrt{2}}$ (D) None of these

33. Two vertices of a triangle are (–2, 1) and (2, 3) and the third vertex moves on the line $3x + 5y = 7$, then the locus of the centroid of the triangle is

A) $5x + 3y = 4$ (B) $3x + 5y + 1 = 0$

(C) $3x + 5y = 1$ (D) None of these

34. If $x,\ 2x + 4,\ 3x + 6$ are in G.P. then fourth term of the sequence is

(A) 25 (B) 32

(C) –27 (D) None of these

35. The ratio of 4^{th} and 11^{th} arithmetic means out of n arithmetic means between –6 and 64 is 2:7. The value of n is

(A) 13 (B) 15

(C) 16 (D) None of these

36. A and B are two events such that $P(\overline{A}) = \frac{2}{3}, P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}$, then $P(\overline{A} \cap B)$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{7}{12}$

(C) $\frac{5}{6}$ (D) $\frac{5}{12}$

37. The fourth term from the end in the expansion of $\left(\frac{5}{2x}-\frac{4x}{5}\right)^9$ is

(A) $\frac{43008}{125}x^3$ (B) $\frac{42568}{125x^3}$

(C) $\frac{46256}{375}x^4$ (D) None of these

38. If three consecutive coefficients in the expansion of $(1+x)^n$ are 936, 585 and 260, then n is equal to

(A) 9 (B) 7

(C) 8 (D) None of these

39. $\sum_{r=2}^{n} {}^nC_r 3^r$ is equal to

(A) 4^n-1 (B) 3^n-2n-1

(C) 4^n-3n-1 (D) None of these

40. The middle term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^8$ is

(A) $5280x^3$ (B) $5670x^2$

(C) $4260x^2$ (D) None of these

41. The sum of first n terms of the sequence $\frac{5}{7},\frac{2}{3},\frac{13}{21},\frac{4}{7},\ldots$ is 5, then n is equal to

(A) 21 (B) 18

(C) 24 (D) None of these

42. If $f(x)=\frac{bx-a}{ax-b}=y,$ then $f(y)$ is equal to

(A) $1-x$ (B) $x-1$

(C) $-x$ (D) None of these

43. A vertical line can meet the graph of a function

(A) in infinite number of points (B) at exactly two points

(C) in no point (D) in not more than one point

44. $\sum_{i=1}^{18} \cos^2\left(5n^o\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{17}{2}$

(C) 1 (D) None of these

45. The general value of x satisfying the equation $\sec x + \tan x = \sqrt{2}\tan x + 1$ is

(A) $2n\pi + \frac{\pi}{4}, n \in Z$ (B) $n\pi + \frac{\pi}{4}, n \in Z$

(C) $2n\pi - \frac{\pi}{4}, n \in Z$ (D) None of these

46. The argument $\bar{z}$ if the complex number $z = \frac{-3+i}{2+i}$ is

(A) $\frac{3\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $-\frac{3\pi}{4}$ (D) None of these

47. If $f\left(x\right) = x^2 - 4x,$ then $f\left(x+1\right) - f\left(x-1\right)$ is equal to

(A) $4(x-2)$ (B) $4(2-x)$

(C) $4(x+2)$ (D) None of these

48. The solution set of the inequality $2x^2 - x < 1$, is

(A) $\left(-\frac{3}{2}, 2\right)$ (B) $\left(-\infty, -\frac{1}{2}\right) \cup \left(1, \infty\right)$

(C) $\left(-\frac{1}{2}, 1\right)$ (D) None of these

49. If A = {1, 2, 3, 4}, B = {x, y, z}, then number of functions that can be defined from A into B is

(A) 64 (B) 81

(C) 2^{12} (D) None of these

50. If $\left(n+3\right)! = 2184\left(n!\right)$, then n is equal to

(A) 7 (B) 10

(C) 12 (D) 11

2 MARKS EACH (51 – 75)

51. The angles of a triangle are in A.P. and the number of degrees in the smallest angle to the number of radians in the largest angle is as $52:\pi$. The measurement of the largest angle is

(A) 80^o (B) 96^o

(C) 92^o (D) None of these

52. If $\sin^2 x - 3\cos x - 2 = 0$, then the value of $\cos^3 x + \sec^3 x$ is

(A) –36 (B) 18

(C) 24 (D) None of these

53. A complex number when divided by (3–7i) gives the inverse of (2+5i), then the inverse of that complex number is

(A) $-\frac{3}{2}+\frac{1}{2}i$ (B) $-\frac{1}{2}+\frac{3}{2}i$

(C) $-\frac{1}{2}+\frac{1}{2}i$ (D) $-\frac{1}{2}-\frac{1}{2}i$

54. The complex root of the equation $x^2 + 2ix = 3\sqrt{2}\,x + 6\sqrt{2}\,i$ is

(A) $1-i$ (B) $i-1$

(C) $-i$ (D) $-2i$

55. The sum of first n terms of the series: $\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+...$

(A) $2^{-n-1}+n-1$ (B) $2^{-n-1}+n-\frac{1}{2}$

(C) $2^{-n}+n$ (D) None of these

56. If $\sqrt{3}\cos ecx - \sec x - 4 = 0$, then the value of x is

(A) 20^o (B) 36^o

(C) 18^o (D) None of these

57. If $x = 2 + 5i$, then the value of $x^3 - 5x^2 + 33x - 21$ is

(A) 12 (B) 18

(C) 8 (D) None of these

58. The second, third and fourth terms in the expansion of $(x+a)^n$ are 810, 1080 and 720 respectively. The value of $(n+x)$ is

(A) 7 (B) 10

(C) 9 (D) 8

59. The number of all possible arrangements of the letters of the word 'ARYABHATTA', taking 4 letters at a time, is

(A) 769 (B) 627

(C) 675 (D) None of these

60. The coefficient of x^2 in the expansion of $\left(1+\frac{1}{x}\right)^8\left(1-2x^3+3x^5\right)$ is

(A) 246 (B) 152

(C) 198 (D) None of these

61. The sum of the series: $\frac{1}{3}+\frac{1}{2}+\frac{3}{4}+\ldots+\frac{243}{64}$, is

(A) $\frac{2257}{288}$ (B) $\frac{2365}{242}$

(C) $\frac{2059}{192}$ (D) None of these

62. The value of $\left(\sqrt{2}+\sqrt{5}\right)^6-\left(\sqrt{2}-\sqrt{5}\right)^6$ is

(A) $718\sqrt{10}$ (B) $208\sqrt{10}$

(C) $516\sqrt{10}$ (D) None of these

63. If the ratio of 5th term from the beginning to the 5th term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $1:\sqrt{6}$. The value of n is

(A) 8 (B) 10

(C) 6 (D) None of these

64. The sum of $(n+1)$ terms of the series: 7+77+777+7777+ . . ., is

(A) $\frac{7}{81}\left(10^{n+1}-9n-11\right)$ (B) $\frac{7}{81}\left(10^{n+2}-9n-19\right)$

(C) $\frac{7}{81}\left(10^{n}-9n-9\right)$ (D) None of these

65. The sum of the series: $3^2 + 5^2 + 7^2 + ...$ to n terms , is

(A) $\frac{n}{3}\left(4n^2 + 12n + 11\right)$ (B) $\frac{n}{3}\left(3n^2 + 5n - 8\right)$

(C) $\frac{n}{3}\left(4n^2 - 12n - 11\right)$ (D) None of these

66. The coordinates of the incentre of the triangle whose vertices are (3, 4), (3, 1) and (7, 1) are

(A) (2, 3) (B) (4, 3)

(C) (2, 2) (D) (4, 2)

67. The triangle formed by the lines: $y + 2 = 0, 7x + 2y - 10 = 0$ and $7x - 2y + 10 = 0$, is

(A) an equilateral triangle (B) a right triangle

(C) an isosceles triangle (D) None of these

68. The equation of the parabola whose focus is (1, 3) and directrix is the line $x + y = 0$, is

(A) $x^2 + y^2 - 2xy + 12 = 0$ (B) $x^2 + y^2 - 4x - 12y - 2xy + 20 = 0$

(C) $x^2 + y^2 - 2x - 6y - 2xy + 10 = 0$ (D) None of these

69. The mean and variance of 8 observations are 10 and 9.25. If six of the observations are 5, 7, 9, 11, 13, 14, then the product of the missing terms is

(A) 104 (B) 84

(C) 88 (D) None of these

70. The image of the point (3, 6) with respect to the line $3x + 2y - 8=0$, assuming the line to be a plane mirror, is

(A) (2, 3) (B) (–3, 2)

(C) (3, 8) (D) None of these

71. The radius of the circumcircle of the triangle whose vertices are (5, –6), (1, 2) and (3, –4) is

(A) 12 units (B) 14 units

(C) 8 units (D) 10 units

72. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{2}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ (B) $\frac{y}{2\sqrt{x}}$

(C) $\frac{2\sqrt{x} - y}{2x}$ (D) None of these

73. The centre of the circle passing through the points (8, 1), (2, –7) and (9, –6), is

(A) (5, –3) (B) (–3, –2)

(C) (–3, 8) (D) (5, –4)

74. A line passes through the point of intersection of the lines $4x + 3y = 7$ and $3x + y = 9$ and perpendicular to the line $10x – 8y + 5=0$. The point of intersection of the above line and x–axis is

(A) $\left(-2,0\right)$ (B) $\left(\frac{1}{4},0\right)$

(C) $\left(\frac{3}{4},0\right)$ (D) None of these

75. In a school, out of 70 students, 42 students opted for Mathematics, 20 students opted for Physics and 24 students opted for Chemistry. Only 4 students opted for all the three subjects. The number of students who opted for exactly 2 subjects is

(A) 9 (B) 10

(C) 6 (D) 8