Summer Fields School

KAILASH COLONY, NEW DELHI-110048

Roll No.										

A

- Please check that this questionnaire contains **15** printed pages.
- Code A, B or C given on the right hand top corner of the questionnaire should be written on the answer sheet in the space provided.
- Please check that this questionnaire contains **60** questions.

36TH ARYABHATTA INTER-SCHOOL MATHEMATICS COMPETITION – 2019

CLASS - XI

Time Allowed: **2 Hours** Max. Marks: **100**

GENERAL INSTRUCTIONS:

1. Do not write your name on the questionnaire.
2. Write your roll no. on the questionnaire and the Answer Sheet in the space provided.
3. All the questions are compulsory.
4. Read questions carefully; think twice before you write the answer. **No overwriting or cutting is allowed on the Answer Sheet.** Another copy of the questionnaire or answer sheet will not be provided.
5. Do your rough work in the space provided in the questionnaire.
6. The questionnaire contains three sections. **Section A** contains 30 Multiple Choice Questions of 1mark each, **Section B** contains 20 Free Response Type Questions of 2 marks each and **Section C** contains 10 Free Response Type Questions of 3 marks each.
7. No working or descriptive answers of any question is to be given. Only the Answers are to be written on the Separate Answer sheet provided to you.
8. Use Blue or Black pens to write the answer on the Answer Sheet.
9. Answers should be written clearly in the space provided on the Answer sheet.
10. Use of calculator is not allowed.

SECTION-A

Write the correct option (A, B, C or D) in the Answer sheet.

1. The expression $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ can be written as
   (A) $\sin A\cos A+1$ (B) $\sec A\cos ecA+1$
   (C) $\tan A+\cot A$ (D) $\sec A+\cos esA$

2. Which of the following is an empty set?
   (A) $\{x: x \text{ is a real number and } x^2-1=0\}$ (B) $\{x: x \text{ is a real number and } x^2+1=0\}$
   (C) $\{x: x \text{ is a real number and } x^2-9=0\}$ (D) $\{x: x \text{ is a real number and } x^2=x+2\}$

3. Given two finite sets A and B such that n(A) = 2, n(B) = 3. Then total number of relations from A to B is
   (A) 4 (B) 8 (C) 64 (D) None of these

4. The relation R defined on the set A = {1, 2, 3, 4, 5} by R = $\{(x, y) : |x^2-y^2|<16\}$ is given by
   (A) {(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)} (B) {(2, 2), (3, 2), (4, 2), (2, 4)}
   (C) {(3, 3), (3, 4), (5, 4), (4, 3), (3, 1)} (D) None of these

5. 8^{th} term of the series $2\sqrt{2}+\sqrt{2}+0+\ldots$ will be
   (A) $-5\sqrt{2}$ (B) $5\sqrt{2}$ (C) $10\sqrt{2}$ (D) $-10\sqrt{2}$

SPACE FOR THE ROUGH WORK

6. Which term of the sequence (−8+18i), (−6+15i), (−4+12i),..... is purely imaginary?
(A) 5^{th} (B) 7^{th} (C) 8^{th} (D) 6^{th}

7. If the p^{th} term of an A.P. be q and q^{th} term be p, then its r^{th} term will be
(A) p+q+r (B) p+q−r (C) p+r−q (D) p−q−r

8. If $\tan n\theta = \tan m\theta$, then the different values of θ will be in
(A) A.P. (B) G.P. (C) H.P. (D) none of these

9. If m^{th} terms of the series 63+65+67+69+..... and 3+10+17+24+..... be equal, then m=
(A) 11 (B) 12 (C) 13 (D) 15

10. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then the value of $3a^2 - 32a + 99$ is
(A) 20 (B) 15 (C) 12 (D) None of these

SPACE FOR THE ROUGH WORK

11. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:
(A) 3 (B) 5 (C) 7 (D) None of these

12. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half the distance between its foci, is
(A) $\frac{4}{\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $4\sqrt{3}$

13. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + (4)^2 + \left(4\frac{4}{5}\right)^2 + \ldots$ is $\frac{16}{5}m$, then m is equal to:
(A) 102 (B) 100 (C) 101 (D) 99

14. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is
(A) $\frac{4}{3}$ (B) $\frac{8}{5}$ (C) $\frac{2}{3}$ (D) 1

15. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is:
(A) 2187 (B) 64 (C) 243 (D) 729

SPACE FOR THE ROUGH WORK

16. The sum of all real values of x satisfying the equation $\left(x^2-5x+5\right)^{x^2+4x-60}=1$ is:

(A) 3 (B) –4 (C) 6 (D) 5

17. A positive value of sin θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is:

(A) $\frac{1}{2}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) None of these

18. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL are arranged as in a dictionary; then the position of the word SMALL is:

(A) 46^{th} (B) 59^{th} (C) 52^{th} (D) 58^{th}

19. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is:

(A) 6 (B) 10 (C) 5 (D) 20

20. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set A × B, each having at least three elements is:

(A) 219 (B) 256 (C) 2758 (D) 510

SPACE FOR THE ROUGH WORK

21. The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6, 7 and 8, without repetition, is:
(A) 216 (B) 192 (C) 124 (D) 72

22. The circle passing through the point (−1, 0) and touching the y−axis at (0, 2) also passes through the point
(A) $\left(-\frac{3}{2},0\right)$ (B) $\left(-\frac{5}{2},2\right)$ (C) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (D) (−4, 0)

23. The wire in the shape of a circle of diameter 10cm is cut and placed along the circumference of a circle of diameter 1 metre. The angle subtended by the wire at the centre of the circle (in radian) is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{5}$ (D) $\frac{\pi}{10}$

24. The value of $\sin^2 5^o + \sin^2 10^o + \sin^2 15^o + ... + \sin^2 85^o + \sin^2 90^o$ is equal to

(A) 7 (B) 9.5 (C) 8 (D) 9

25. If $3\pi/4<\alpha<\pi$, then $\sqrt{\text{cosec}^2\alpha + 2\cot\alpha}$ is equal to
(A) $1+\cot\alpha$ (B) $1-\cot\alpha$ (C) $-1-\cot\alpha$ (D) $-1+\cot\alpha$

SPACE FOR THE ROUGH WORK

26. The sum of all four digit numbers that can be made using the digits 1, 2, 3, 4 when repetition of digits is not allowed, is

(A) 66660 (B) 36600 (C) 36000 (D) 66000

27. The polar form of the complex number $\sqrt{3}+i$ is

(A) $4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ (B) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

(C) $4\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$ (D) $2\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$

28. If $\underline{\lfloor n+2} = 2550\underline{\lfloor n}$, then n is equal to

(A) 48 (B) 52 (C) 49 (D) 46

29. The eccentricity of the hyperbola $x^2 - y^2 = a^2$ is

(A) $\frac{3}{2}$ (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

30. The value of $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}$ is

(A) $-\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{8}$ (D) $\frac{1}{16}$

SPACE FOR THE ROUGH WORK

SECTION-B

Write the Answers only in the space provided on the Answer sheet.

31. Find the integral value of $n > 3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$.

32. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0$, and S = $\{x \in R : f(x) = f(-x)\}$; then write S in Roster Form.

33. Find the square root of $\left(\frac{86-27i}{2+3i}\right)$.

34. Evaluate: $\lim_{x\to 0}\left(\frac{27^x - 9^x - 3^x + 1}{1-\cos x}\right)$.

SPACE FOR THE ROUGH WORK

35. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture should not be less than 5% and should not be more than 7%. If there is 460 litres of 9% solution, find the difference between the maximum and minimum quantities of 3% acid solution to be added to get the above mixture.

36. Find $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$.

37. Two vertices of a triangle are (5, –1) and (–2, 3) and its orthocenter is at origin. Find the coordinates of the third vertex.

38. 52 cards are distributed equally among 4 players. Find the probability that the four kings are held by a specified player.

SPACE FOR THE ROUGH WORK

39. Find the sum of first 50 terms of the sequence 7, 7.7, 7.77, 7.777, ...

40. Find the coordinates of the incentre of the triangle whose vertices are (–36, 7), (20, 7) and (0, –8).

41. A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

42. The mean and variance of the observations 14, 12, 10, 4, 2, x and y are 8 and 16 respectively. Find the value of $(x-y)^2$.

SPACE FOR THE ROUGH WORK

43. If $x = 3\cos\theta - 2\cos^3\theta$ and $y = 3\sin\theta - 2\sin^3\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

44. Find the equation of a circle with origin as the centre and passing through the vertices of an equilateral triangle whose median is of length 3a.

45. Find $x + y$ if $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$, where $i = \sqrt{-1}$ and $x, y \in R$.

46. Find the solution set of the following system of linear inequations:

$2x - 3 \le 5, \frac{2x+5}{x+7} \ge 3$.

SPACE FOR THE ROUGH WORK

47. Find the sum of the series $\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+...$ upto n terms.

48. If A (1, 2) and B (3, 8) be two given points, find the point P in first quadrant such that $|PA|=|PB|$ and area of the triangle PAB=10 square units.

49. Evaluate $\tan 81^\circ - \tan 63^\circ - \tan 27^\circ + \tan 9^\circ$.

50. The equation of the sides AB, BC, CA of a triangle ABC are $3x+4y=6, 12x-5y=3$ and $4x-3y+12=0$. Find the equation of the internal bisector of angle A.

SPACE FOR THE ROUGH WORK

SECTION-C

Write the Answers only in the space provided on the Answer sheet.

51. In a purse there are 9 coins of ₹1 and 1coin of ₹10. In another purse there are 10 coins of ₹1. 9 coins are taken from the first purse and put in to the second and then nine coins are taken from the second purse and put in to the first. Find the probability that the ₹10 coin is still in the first purse.

52. How many odd numbers are there between 1000 and 10000 which have none of their digits repeated?

53. The sum of coefficients of first three terms in the expansion of $\left(x-\frac{3}{x^2}\right)^n, x \neq 0, n$ being a natural number is 559. Find the coefficient of x^3.

54. The intercepts cut off by a line from y-axis is twice that from x-axis and the line passes through the point (1, 2). Find the equation of the line.

SPACE FOR THE ROUGH WORK

55. Find the sum of all natural numbers amongst first 1000 natural numbers which are neither divisible by 2 nor by 5.

56. Find the sum of the series $1+\frac{12}{5}+\frac{63}{25}+\frac{270}{125}+...$ upto n terms.

57. Find the term independent of x in the expansion of $\left(1+x+2x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$.

SPACE FOR THE ROUGH WORK

58. Find the sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$.

59. If the coefficients of three successive terms in the expansion of $(1+x)^m$ are 220, 495 and 792, then find the value of *m*.

60. The points A (3, 2, 0), B (5, 3, 2) and C (0, 2, 4) are the vertices of the triangle ABC. The bisector of the angle BAC meets BC at D. Find the length of AD.

SPACE FOR THE ROUGH WORK