Summer Fields School

KAILASH COLONY, NEW DELHI-110048

Roll No.										

A

- Please check that this questionnaire contains **15** printed pages.
- Code A, B or C given on the right-hand top corner of the questionnaire should be written on the answer sheet in the space provided.
- Please check that this questionnaire contains **60** questions.

40TH ARYABHATTA INTER-SCHOOL MATHEMATICS COMPETITION – 2023

CLASS - XI

Time Allowed: **2 Hours** Max. Marks: **100**

GENERAL INSTRUCTIONS:

1. Do not write your name on the questionnaire.
2. Write your roll no. on the questionnaire and the Answer Sheet in the space provided.
3. All the questions are compulsory.
4. Read questions carefully; think twice before you write the answer. **No overwriting or cutting is allowed on the Answer Sheet.** Another copy of the questionnaire or answer sheet will not be provided.
5. Do your rough work in the space provided in the questionnaire.
6. The questionnaire contains three sections. **Section A** contains 30 Multiple Choice Questions of 1mark each, **Section B** contains 20 Free Response Type Questions of 2 marks each and **Section C** contains 10 Free Response Type Questions of 3 marks each.
7. No working or descriptive answers of any question is to be given. Only the Answers are to be written on the Separate Answer sheet provided to you.
8. Use Blue pen to write the answer on the Answer Sheet.
9. Answers should be written clearly in the space provided on the Answer sheet.
10. Use of calculator is not allowed.

SECTION-A

Write the correct option (A, B, C or D) in the Answer sheet.

1. Given $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ

   (A) $1 \le A \le 2$ (B) $\frac{1}{16} \le A \le 1$

   (C) $\frac{3}{4} \le A \le 1$ (D) None of these

2. The sides of a right-angled triangle are in arithmetic progression. If the triangle has area 24 square units, then the length of the smallest side is

   (A) 4 (B) 6

   (C) 8 (D) None of these

3. For any three real positive numbers a, b and c,
   $9\left(25a^2 + b^2\right) + 25\left(c^2 - 3ac\right) = 15b\left(3a + c\right)$. Then

   (A) a, b and c are in G.P. (B) b, c and a are in G.P.

   (C) b, c and c are in A.P. (D) a, b and c are in A.P.

4. Let a vertical tower AB has its end A on the level ground. Let C be the midpoint of AB and P be the point on the ground such that AP = 2 AB. If $\angle$ BPC = θ, then $\tan\theta$ is equal to

   (A) $\frac{4}{7}$ (B) $\frac{6}{7}$

   (C) $\frac{2}{9}$ (D) None of these

5. The range of the function $f(x) = \frac{1}{2 - \sin 3x}$ is

   (A) $\left[\frac{1}{3}, 1\right]$ (B) $\left(\frac{-1}{2}, 1\right)$

   (C) $\left[\frac{-1}{3}, 1\right]$ (D) None of these

SPACE FOR THE ROUGH WORK

6. The value of $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$ is equal to

(A) 2 (B) 6

(C) 4 (D) None of these

7. The solution of the system of inequalities $\frac{x}{2x+1}\geq\frac{1}{4}, \frac{6x}{4x-1}<\frac{1}{2}$, is

(A) $-\frac{1}{2}\leq x<\frac{1}{2}$ (B) $-\frac{1}{8}<x<\frac{1}{4}$

(C) $-\frac{1}{8}\leq x<\frac{1}{2}$ (D) None of these

8. If $\frac{\pi}{2}<x<\pi$, then $\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ is equal to

(A) $\tan x$ (B) $-\tan x$

(C) $\cot x$ (D) None of these

9. The coordinates of the point which is equidistant from the points O (0, 0, 0), A (a, 0, 0), B (0, b, 0) and C (0, 0, c) are

(A) $\left(\frac{a}{2},0,0\right)$ (B) $\left(0,\frac{b}{2},0\right)$

(C) $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$ (D) $\left(0,0,\frac{c}{2}\right)$

10. If $y=\frac{\sec x+\tan x}{\sec x-\tan x}$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{6}$ is

(A) $4\sqrt{3}$ (B) 2

(C) 1 (D) None of these

SPACE FOR THE ROUGH WORK

11. $\lim\limits_{x\to\pi}\left(\dfrac{1-\sin\dfrac{x}{2}}{\cos\dfrac{x}{2}\left(\cos\dfrac{x}{4}-\sin\dfrac{x}{4}\right)}\right)=$

(A) $2\sqrt{2}$ (B) $-\sqrt{2}$

(C) $\sqrt{2}$ (D) None of these

12. The ratio in which the line segment joining the points (2, 4, 5) and (3, 5, –9) is divided by YZ-plane is

(A) 2 : 3 (B) 3 : 2

(C) –2 : 3 (D) None of these

13. The value of $\cos^6 x+\sin^6 x+3\sin^2 x-3\sin^4 x$ is equal to

(A) 1 (B) 2

(C) 0 (D) None of these

14. The polar form of $\left(i^{25}\right)^3$ is

(A) $\sqrt{2}\left(\cos\dfrac{\pi}{2}-i\sin\dfrac{\pi}{2}\right)$ (B) $\cos\dfrac{\pi}{2}-i\sin\dfrac{\pi}{2}$

(C) $-\cos\dfrac{\pi}{2}+i\sin\dfrac{\pi}{2}$ (D) None of these

15. If $\tan 66^\circ+\tan 69^\circ-\tan 66^\circ\tan 69^\circ=4k$, then the value of k is

(A) $-\dfrac{1}{4}$ (B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{2}$ (D) None of these

SPACE FOR THE ROUGH WORK

16. The value of $1.1!+2.2!+3.3!+\ldots+n.n!$ is

(A) $(n+1)!$ (B) $(n+2)!$

(C) $n!$ (D) None of these

17. The range of the function $f(x) = {}^{7-x}P_{x-3}$ is

(A) $\{3,4,5,6,7\}$ (B) $\{1,2,3\}$

(C) $\{1\}$ (D) None of these

18. The coefficient of $a^{10}b^7c^3$ in the expansion of $(ab+bc+ca)^{10}$ is

(A) 30 (B) 60

(C) 120 (D) None of these

19. A dice is thrown (2*n*+1) times. The probability that the faces with even numbers appear odd number of times, is

(A) $\frac{2n+1}{2n+3}$ (B) $\frac{n+1}{2n+1}$

(C) $\frac{n}{2n+1}$ (D) None of these

20. Let *x* be an integer such that the triangle with vertices (5, *x*), (–*x*, 2) and (*x*, –3*x*) has area 28 sq. units. Then the orthocenter of this triangle is at the point

(A) $\left(2,\frac{1}{2}\right)$ (B) $\left(2,-\frac{1}{2}\right)$

(C) $\left(2,\frac{3}{4}\right)$ (D) None of these

SPACE FOR THE ROUGH WORK

21. The coordinates of the foot of the perpendicular drawn from a point (3, 4, 5) on the XY-plane are
(A) (3, 4, 0) (B) (0, 0, 5)
(C) (3, 0, 0) (D) None of these

22. The smallest positive integer m for which $\left(\frac{1+i}{1-i}\right)^m = 1$ is
(A) 8 (B) 12
(C) 16 (D) None of these

23. A rod AB of length 15 cm rests between two coordinate axes in such a way that point A lies on x-axis and B lies on y-axis. A point P is taken on the rod in such a way that AP = 6 cm. The locus of the point P is
(A) $2x + 3y = 16$ (B) $4x^2 + 9y^2 = 324$
(C) $x^2 + y^2 = 36$ (D) None of these

24. The complex numbers $z = x+iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lie on
(A) the x-axis (B) the line y =5
(C) A circle passing through the origin (D) None of these

25. The radius of the circle represented by the equation $3x^2 + 3y^2 + kxy + 9x + (k-6)y + 3 = 0$ is
(A) 2 (B) 1.5
(C) 1 (D) None of these

SPACE FOR THE ROUGH WORK

26. The foci of the hyperbola coincide with the foci of the ellipse $9x^2 + 25y^2 = 225$, the equation of the hyperbola if the eccentricity is 2, is

(A) $x^2 - 4y^2 = 8$ (B) $3x^2 - y^2 = 27$

(C) $3x^2 - y^2 = 12$ (D) None of these

27. If A and B are mutually exclusive events of a sample space and P (A) = 0.3, P (B) = 0.6 then $P\left(\overline{A} \cap \overline{B}\right)$ is

(A) 0.91 (B) 0.1

(C) 0.9 (D) None of these

28. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is

(A) $x = -1$ (B) $x = 2$

(C) $x = 1.5$ (D) None of these

29. The probability of getting a multiple of 2 or a multiple of 9 from 1 to 1000 is

(A) $\frac{139}{250}$ (B) $\frac{143}{250}$

(C) $\frac{249}{500}$ (D) None of these

30. The solution set of $\frac{|x-2|}{x-2} \geq 0$ is

(A) $(0, \infty)$ (B) $(2, \infty)$

(C) $[2, \infty)$ (D) None of these

SPACE FOR THE ROUGH WORK

SECTION-B

Write the Answers only in the space provided on the Answer sheet.

31. Two sets each of 20 observations have the same standard deviation 5. The first set has the mean 17 and the second set has the mean 22. Write the standard deviation of the set of observations obtained by combining the given two sets.

32. Write the General solution of $\sin 3x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

33. Write the equation of the straight line passing through the point (3, –4) and cutting off intercepts, equal but of opposite signs, from the two axes.

34. If $\overline{z} = x + iy$ and $z^{\frac{1}{3}} = p + iq$, then write the value of $\dfrac{\left(\dfrac{x}{p} + \dfrac{y}{q}\right)}{\left(p^2 + q^2\right)}$.

SPACE FOR THE ROUGH WORK

35. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then write the sum of the coefficients of all the terms in this expansion.

36. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then write the common ratio of the G.P.

37. Write the incentre of the triangle whose vertices are $\left(1,\sqrt{3}\right),(2,0)$ and $(0,0)$.

38. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, then write the equation of the reflected ray.

SPACE FOR THE ROUGH WORK

39. Six cards and six envelopes are numbered 1, 2, 3, 4, 5, 6 and cards are to be placed in envelopes so that each envelope contains exactly one card and no card is placed in the envelope bearing the same number and moreover the card numbered 1 is always placed in the envelope numbered 2. Write the number of ways it can be done.

40. If the real part of $\frac{\overline{z}+2}{\overline{z}-1}$ is 4, then write the equation of the locus of the point representing $z = x + iy$ in the complex plane.

41. If a parallelepiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to coordinate planes then write the length of the diagonal of the parallelepiped.

42. Write the value of $\lim\limits_{x\to\frac{\pi}{6}}\left(\frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}\right)$.

SPACE FOR THE ROUGH WORK

43. Write the equation of the Hyperbola whose foci are (8, 3) and (0, 3) and eccentricity is $\frac{4}{3}$.

44. If $x^m.y^n = (x+y)^{m+n}$, write $\frac{dy}{dx}$ in terms of x and y only.

45. The 3rd, 4th and 5th terms in the expansion of $(x+a)^n$ are respectively 84, 280 and 560. Write the value of $(n+a)$.

46. Write the number of words that can be formed by taking 4 letters of the word 'MURADABAD'.

SPACE FOR THE ROUGH WORK

47. Write the principal argument of the complex number $\dfrac{-i-1}{\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}}$.

48. Write the general solution of the equation $\tan x+\tan\left(x+\dfrac{\pi}{3}\right)+\tan\left(x+\dfrac{2\pi}{3}\right)=3.$

49. Write the value of $\sec\dfrac{\pi}{9}-\sqrt{3}\cos ec\dfrac{\pi}{9}.$

50. If ${}^{n-2}\mathrm{P}_4 : {}^{n+2}\mathrm{C}_8 = 16:57,$ write the value of n.

SPACE FOR THE ROUGH WORK

SECTION-C

Write the Answers only in the space provided on the Answer sheet.

51. Write the sum of the coefficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$.

52. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then write the equation of the locus of R.

53. Harshit has 7 friends, 4 of them are ladies and 3 are men. His wife Anu also has 7 friends, 3 of them are ladies and 4 are men. Assume Harshit and Anu have no common friends. Write the total number of ways in which Harshit and Anu together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of them are in this party.

54. Write the coefficient of y^{99} in the polynomial $(y-1)(y-2)\ldots(y-100)$.

SPACE FOR THE ROUGH WORK

55. If $\cos x = \frac{1}{5}$, write the value of $\frac{\cos 5x + \cos 4x}{1 - 2\cos 3x}$.

56. Three girls and two boys stand in a queue. Write the probability that the number of girls ahead of every boy is at least one more than the number of boys ahead of him.

57. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English Dictionary. Write the 100^{th} word in the above order.

SPACE FOR THE ROUGH WORK

58. If the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$ is equal to the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$, then write the value of $(ab + 2)$.

59. If $z_1 = 10+6i, z_2 = 4+6i$ and z is any complex number such that $\arg\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$, then write the value of $|z-7-9i|$.

60. Let P be any point on the parabola $y^2 = 8x$ which is at a minimum distance from the centre C of the circle $x^2 + y^2 + 12y + 35 = 0$. Write the equation of the circle with centre at P and passing through C, in general form.

SPACE FOR THE ROUGH WORK