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A

- Please check that this questionnaire contains **15** printed pages.
- Code A, B or C given on the right hand top corner of the questionnaire should be written on the answer sheet in the space provided.
- Please check that this questionnaire contains **60** questions.

41st ARYABHATTA INTER-SCHOOL MATHEMATICS COMPETITION – 2024

CLASS - XI

Time Allowed: **2 Hours**

Max. Marks: **100**

GENERAL INSTRUCTIONS:

1. Do not write your name on the questionnaire.
 2. Write your roll no. on the questionnaire and the Answer Sheet in the space provided.
 3. All the questions are compulsory.
 4. Read questions carefully; think twice before you write the answer. **No overwriting or cutting is allowed on the Answer Sheet.** Another copy of the questionnaire or answer sheet will not be provided.
 5. Do your rough work in the space provided in the questionnaire.
 6. The questionnaire contains three sections. **Section A** contains 30 Multiple Choice Questions of 1 mark each, **Section B** contains 20 Free Response Type Questions of 2 marks each and **Section C** contains 10 Free Response Type Questions of 3 marks each.
 7. No working or descriptive answers of any question is to be given. Only the Answers are to be written on the Separate Answer sheet provided to you.
 8. Use Blue or Black pens to write the answer on the Answer Sheet.
 9. Answers should be written clearly in the space provided on the Answer sheet.
 10. Use of calculator is not allowed.
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SECTION-A

Write the correct option (A, B, C or D) in the Answer sheet.

1. In a class of 50 students, 19 like Mathematics, 30 like Physics and 10 like Mathematics but not Physics. Find the number of students who do not like any of the two subjects.
(A) 9 (B) 10 (C) 11 (D) 12
2. The distance between the incentre and the orthocentre of the triangle with the vertices (1, 2), (2, 1) and $\left(\frac{3+\sqrt{3}}{2}, \frac{3+\sqrt{3}}{2}\right)$ is
(A) $\sqrt{3}$ (B) 0 (C) 2 (D) None of these
3. How many numbers are there between 100 and 1000 which have exactly one of their digits as 5?
(A) 250 (B) 256 (C) 225 (D) 234
4. If $f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ [x], & \text{when } 0 \leq x < 2, \\ x^3 - 2, & \text{when } x \geq 2 \end{cases}$ find $f(2) + f(\sqrt{2}) - f(-2)$.
(A) 4 (B) 2 (C) 6 (D) 3
5. The number of terms in the expansion of $(1 - 6x + 12x^2 - 8x^3)^{10}$
(A) 11 (B) 40 (C) 61 (D) 31

SPACE FOR THE ROUGH WORK

6. If sets A and B are defined as $A = \{(x, y) : y = -x, x \in R\}$, $B = \{(x, y) : y = \frac{1}{x}, 0 \neq x \in R\}$, then
 (A) $A \cap B = A$ (B) $A \cap B = B$ (C) $A \cap B = \phi$ (D) $A \cup B = A$
7. If in an A.P., the p^{th} term q and $(p+q)^{\text{th}}$ term be 0, then its q^{th} term will be
 (A) p (B) $-p$ (C) $p+q$ (D) $p-q$
8. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 88m when it has traced 72° at the centre, the length of the rope is
 (A) 77 metres (B) 56 metres (C) 84 metres (D) 70 metres
9. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that $3 P(A) = 2 P(B) = P(C)$, then $P(B)$ is equal to
 (A) $\frac{3}{11}$ (B) $\frac{2}{7}$ (C) $\frac{5}{11}$ (D) $\frac{3}{7}$
10. The equation $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$ represents
 (A) a circle (B) an ellipse (C) a hyperbola (D) a parabola

SPACE FOR THE ROUGH WORK

11. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person get at least one ball is
(A) 125 (B) 205 (C) 242 (D) 150
12. The sum of an infinite G.P. is 4 and the sum of the cubes of its terms is $\frac{64}{7}$. The common ratio of the original G.P. is
(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$
13. If A, B, C are in A.P. then $\frac{\cos A - \cos C}{\sin A - \sin C}$
(A) $\tan B$ (B) $\cot B$ (C) $-\tan B$ (D) $-\cot B$
14. A school awarded 35 prizes in football, 17 in Basketball and 21 in Cricket. If these prizes went to a total of 60 students and only 3 students got prizes in all the three sports, then the number of persons getting prize in exactly one of the sports is
(A) 46 (B) 50 (C) 48 (D) 51
15. The length of the median AD of the triangle ABC, where D (3, 2, 0), E (3, 0, 3) and F (0, 2, 3) are the mid points the sides BC, AC and AB respectively
(A) 7 (B) 6 (C) $\sqrt{34}$ (D) 8

SPACE FOR THE ROUGH WORK

16. For a positive integer n , the value of $(1-i)^n \left(1-\frac{1}{i}\right)^n$ is 256. The value of n is
(A) 8 (B) 16 (C) 4 (D) 5
17. The mean deviation about the mean of first 16 even natural numbers is
(A) 2.5 (B) 4 (C) 4.5 (D) None of these
18. The value of $\frac{d}{dx} \left(\frac{1}{\sqrt{\tan x}} \right)$ at $x = \frac{\pi}{4}$ is
(A) 1 (B) 2 (C) -2 (D) -1
19. $\lim_{x \rightarrow 0} \frac{\log(2+x) - \log(2-x)}{x}$ is
(A) 0.25 (B) 0.5 (C) 1 (D) None of these
20. If $z = \frac{1}{(2+3i)^2}$, then $|z|$ is equal to
(A) $\frac{1}{13}$ (B) $\frac{1}{12}$ (C) $\frac{1}{5}$ (D) $\frac{1}{2}$

SPACE FOR THE ROUGH WORK

21. At 3:40, the angle between the minute hand and the hour hand is
(A) $\frac{5\pi}{12}$ (B) $\frac{13\pi}{18}$ (C) $\frac{3\pi}{4}$ (D) $\frac{2\pi}{3}$
22. A rope of 83cm is to be cut into three pieces. The second piece is to be 3 cm longer than the shortest and the third piece is to be twice as long as the shortest. The maximum length of the shortest piece is
(A) 22 cm (B) 28 cm (C) 20 cm (D) 24 cm
23. For the first 10 positive integers, if we multiply each number by -1 and then add 1 to each number, the variance of the numbers so obtained is
(A) 8.25 (B) 3.8 (C) 2.8 (D) 6.5
24. If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then $\tan x$ is equal to
(A) $\frac{44}{117}$ (B) $\frac{15}{16}$ (C) $\frac{11}{12}$ (D) $\frac{22}{21}$
25. The domain of the function $f(x) = \frac{1}{\sqrt{x-|x|}}$ is
(A) $(0, \infty)$ (B) $(-1, \infty)$ (C) $(-\infty, 0)$ (D) None of these

SPACE FOR THE ROUGH WORK

26. The maximum value of $3\cos x + 5\sin\left(x - \frac{\pi}{6}\right)$ is
(A) $\sqrt{34}$ (B) 5 (C) $\sqrt{19}$ (D) None of these
27. If ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 512$, then ${}^{2n}C_2$ is equal to
(A) 120 (B) 153 (C) 226 (D) 46
28. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, then $\tan(2A+B)$ is equal to
(A) 4 (B) 3 (C) 2 (D) 1
29. $\lim_{x \rightarrow 0} \frac{x^2 \cos x}{1 - \cos x}$ is equal to
(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) 2 (D) 1
30. The coefficient of variation of two distributions are 50 and 60 and their arithmetic means are 30 and 25 respectively. The difference between their standard deviations is
(A) 0 (B) 1.5 (C) 2 (D) 1
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SPACE FOR THE ROUGH WORK

SECTION-B

Write the Answers only in the space provided on the Answer sheet.

31. Find the domain of the function $f(x) = \sqrt{5|x| - x^2 - 6}$.
32. Find the coefficient of x^5 in the expansion of $(1 - x + x^2)^4$.
33. The words with five letters are to be formed from ten given letters of English alphabets. Find the number of words which have at least one letter repeated.
34. The sum of two numbers is $\frac{13}{6}$. An even number of arithmetic means are being inserted between them and their sum exceeds their number by 1. Find the number of means inserted.

SPACE FOR THE ROUGH WORK

35. Find $\lim_{x \rightarrow \sqrt{2}} \frac{\sqrt{3+2x} - \sqrt{2} - 1}{x^2 - 2}$.

36. Find the value of $\cot 9^\circ - \cot 27^\circ - \cot 63^\circ + \cot 81^\circ$.

37. Six boys and six girls sit in a row randomly. Find the probability that all the girls sit together.

38. If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4$.

SPACE FOR THE ROUGH WORK

39. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$, find r .
40. Find the sum of an infinitely decreasing G.P. whose first term is $b + 2$ and the common ratio is $\frac{2}{c}$, where b is the least value of the product of the roots of the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$, and c is the greatest value of the sum of its roots.
41. Find the range of the function $f(x) = \frac{x^2}{1+x^2}$.
42. If x is the A.M. and y and z are the two G.M.s between two positive numbers, then find the value of $\frac{xyz}{y^3 + z^3}$.

SPACE FOR THE ROUGH WORK

43. Find the most general value of x satisfying the equations $\tan x = -1$ and $\cos x = \frac{\sqrt{2}}{2}$.
44. Find the focus of the conic section given by the equation $4y^2 + 12x - 20y + 67 = 0$.
45. Find the value of $\cos 510^\circ \cos 330^\circ + \sin 390^\circ \cos 120^\circ$.
46. Find a complex number z satisfying the equation $z + \sqrt{2}|z+1| + i = 0$.

SPACE FOR THE ROUGH WORK

47. Find the term independent of x in the expansion of $\left(3x^2 - \frac{1}{2x^3}\right)^{10}$.
48. Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$.
49. A line $4x + y = 1$ through the point A (2, -7) meets the line BC whose equation is $3x - 4y + 1 = 0$ at the point B. Find the equation to the line AC so that $AB = AC$.
50. Six new students, two of whom are siblings, are to be allotted six seats in a row. If the allotment of seats is made randomly, what is the probability that the siblings will have non adjacent seats?

SPACE FOR THE ROUGH WORK

SECTION-C

Write the Answers only in the space provided on the Answer sheet.

51. Find the coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$.
52. Find the equation of the circle circumscribing the triangle formed by the lines $2x+y=4$, $x+y=6$ and $x+2y=5$.
53. Find the equation of the set of all points, the sum of whose distances from the points $(3, 0)$ and $(9, 0)$ is 12.
54. If $f(x) = \frac{\cos x + \sin x}{\cos x - \sin x}$, then find the derivative of $f(x)$ at $x = \frac{7\pi}{12}$.

SPACE FOR THE ROUGH WORK

55. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the square is 8 cm, find the sum of the areas of all the squares so formed.
56. Solve: $\frac{|x+3|+x}{x+2} > 1$.
57. A line forms a triangle of area $54\sqrt{3}$ square units with the coordinate axes. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of 30° with the X-axis.

SPACE FOR THE ROUGH WORK

58. Find the number of words that can be formed by taking 4 letters of the word 'MATHEMATICIAN'.
59. The sum of coefficients of first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$, n being a natural number is 376. Find the coefficient of x^4 .
60. The equations of two sides of a triangle are $3x - 2y + 6 = 0$ and $4x + 5y - 20 = 0$ and the orthocentre is $(1, 1)$. Find the equation of the third side.

SPACE FOR THE ROUGH WORK
